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A STUDY OF LAMINAR BUOYANT JETS DISCHARGED AT AN INCLINATION TO THE VERTICAL BUOYANCY FORCE

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Abstract—A detailed experimental investigation has been carried out to study the trajectory and other thermal characteristics of a buoyant jet discharged at an inclination to the vertical buoyancy force. A jet of hot water is injected, horizontally and at various inclinations to the horizontal, into a water body which simulates an extensive isothermal ambient medium. The temperature distribution in the jet is determined and the trajectory of the jet, under the action of the buoyancy force and the momentum input, is studied as a function of the inflow velocity and temperature. For inclined jets discharged downwards, the study determines the downward penetration of the buoyant jet. The decay of the centerline temperature in the jet and the increase in the spread of the jet downstream are measured. The experimental results are obtained for various values of the mixed convection parameter, which is found to correlate the observed trends very well. The results are also considered in terms of the basic physical mechanisms that govern the flow and a simple analytical model, based on the integral approach, is developed to generalize the experimental observations. The predicted trajectory and the decay of centerline temperature are found to agree quite well with the experimental results.

NOMENCLATURE

- A, ratio of the spread of the thermal field to that of the flow field, equation (3);
- C_p, specific heat of the fluid;
- D, inner diameter of the jet inlet tube;
- g, magnitude of the gravitational acceleration;
- Gr, Grashof number, defined in equation (1);
- Pr, Prandtl number of the fluid;
- Q, volume flow rate at the inlet;
- r, radial distance from jet axis;
- R, width of jet, defined in equation (3);
- Re, Reynolds number, defined in equation (2);
- S, distance along the jet axis;
- t, local temperature;
- t_c, centerline temperature;
- t_i, inlet temperature of the jet;
- t_∞, ambient fluid temperature;
- u, local velocity in the jet flow;
- U_c, centerline velocity;
- U_i, inlet velocity;
- X, horizontal distance from jet inlet;
- Y, vertical upward distance from center of jet inlet.

Greek symbols

- α, entrainment coefficient, defined in equation (4);
- β, coefficient of thermal expansion of the fluid;
- δ, downward penetration distance of a jet inclined downward with the horizontal at inlet;
- ν, kinematic viscosity of the fluid;
- ϕ, dimensionless temperature, defined in equation (1);
- θ, angle of inclination of jet axis with the horizontal.

Superscript

- $'$, dimensionless variable.

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INTRODUCTION

THE THERMAL characteristics of a jet of heated fluid discharged into an extensive isothermal medium of the same fluid are of interest in several heat rejection and energy storage processes. Such buoyant jets are encountered in thermal discharges from power plants and from various industrial systems into water bodies, in effluents from chimneys and cooling towers, in water tanks for energy storage employing heated discharges, in heat removal systems that use liquid recirculation and in buoyant wakes resulting from the convective cooling of heated bodies. Though turbulent jets are of interest in most environmental studies, related to thermal discharge into the ambient air or water medium, laminar buoyant jets are often encountered in industrial systems and in the laboratory. Both laminar and turbulent jets have received considerable attention in the past, with particular emphasis on turbulent jets in the recent years.

Abraham [1] analyzed the gross behavior of turbulent buoyant jets discharged horizontally and compared his results, based on the integral method, with experimental measurements. Turner [2] studied experimentally the flow of turbulent jets of heavy salt solution injected upwards into a tank of fresh water and measured the penetration distance of salt water. Jen *et al.* [3] determined experimentally the characteristics of jets discharged horizontally at the surface of a water body and studied the temperature decay along the jet axis. The laminar vertical buoyant jet was studied by Brand and Lahey [4], who employed the similarity method. They attempted to relate thermal plume and buoyant jet flows. Mollendorf and Gebhart [5] investigated the effect of a small amount of thermal buoyancy in round laminar vertical jets, employing perturbation methods. The results, which were obtained numerically, indicated an increase in the vertical velocity component and a decrease in the jet diameter because of the added buoyancy effect. Various other studies have been carried out recently on buoyant jet flows, such as those by Pryputniewicz and Bowley [6], who studied turbulent buoyant vertical jets, and by Seban *et al.* [7], who measured centerline temperatures and penetration distances in a heated air jet discharged downwards into an ambient air medium. Jaluria [8] discusses various experimental and analytical studies done on such free boundary flows.

An important aspect of buoyant jets that has not received close attention in the literature is that of the jet trajectory when the buoyant jet is not discharged vertically, particularly for laminar jets. The jet axis is a curve in this case since the vertical buoyancy force is not aligned with the direction in which the jet is discharged. This trajectory and the decay of the temperature in the jet downstream are of interest in the design of heat rejection and energy storage systems [9, 10]. Bosarquet *et al.* [11] studied this problem with a photographic visualization technique and developed a theoretical model for analyzing the flow. Anwar [12] and Fan [13] studied the trajectories for positively and negatively buoyant jets, employing fresh and salt water for the density difference. The present work concentrates on the thermal characteristics of laminar buoyant jet flows in which the direction of discharge is at an inclination with the vertical buoyancy force. Interest lies in the jet trajectory, in the spread of the jet downstream, in the temperature decay along the jet axis and in the temperature distributions that arise in the flow. A detailed investigation has been carried out experimentally to determine the dependence of these physical jet variables on the inflow jet velocity, inlet temperature and the angle of inclination of the discharge direction with the vertical, considering two angles of inclination besides the horizontal discharge.

It is found that the jet trajectory, as well as other thermal characteristics of the jet, depend strongly on the mixed convection parameter Gr/Re^2, which compares the natural convection effects with the forced flow mechanisms. A discharge at a downward inclination with the horizontal results in an initial downward movement of the jet, followed by an upward movement. The measured temperature distributions in the jet are found to be quite symmetrical about the jet axis, which is indicated by the peak in the profiles obtained. On the basis of the experimental results obtained, a simple analytical model, employing integral methods, is developed. The laminar jet flow under consideration is analyzed and results are obtained on the jet trajectory, downstream decay of the centerline temperature and spread of the flow for various values of the mixed convection parameter. The analytical results are compared with the measurements made and a fairly good agreement is observed, lending support to such an analytical model for these flows and also consolidating the experimental results in terms of the governing mechanisms and parameters.

EXPERIMENTAL ARRANGEMENT

The experiment simulates the flow of a buoyant jet in an extensive isothermal medium by ensuring that the temperature in the region away from the jet remains unchanged during the experiment. The buoyant water jet, heated by means of a constant temperature heating bath, was discharged into the water contained in a Plexiglas tank of inside dimensions 0.6 × 0.6 × 0.44 m. The tank was insulated on the sides with Thermocole sheets of thickness 0.25 m. On one side of the tank, three circular holes were drilled in a vertical line along the middle and brass tubes of I.D. 0.6 cm were inserted in these holes, with proper sealing against leakage. These openings were employed in the discharge of the heated water into the tank.

Water from an overhead tank flows through a coiled copper tube placed inside the temperature bath. The temperature of the bath could be varied and the hot water flow rate adjusted by means of a valve arrangement. A rotameter was fitted at one end of the tube, through which the hot water was allowed to flow and which was well-insulated, in order to measure the flow rate. The hot water leaving the coil was discharged into the water body at any one of the desired inlet locations, the bottom and the middle, or center, locations being employed in this study, since the top one was too near the surface of water to simulate an extensive environment. The temperatures at the inlet into the tank and in the water body were measured by the use of copper–constantan thermocouples, employing a 48-channel Honeywell temperature recorder with an accuracy of about 0.15°C. The thermocouples were calibrated using an oil bath. A support was made for ten thermocouple junctions located vertically. The support could be moved horizontally, as well as vertically, so as to determine the temperature distribution in the water body, particularly in the region of the buoyant jet. Care was taken to keep the probe support well outside the jet region [14].

The hot water flow rates were kept small so that laminar flow in the inlet tube was obtained. The level of water in the tank rose during the experiment, but the rise was quite small, as compared to the vertical distance between the jet inlet and the surface of water, for the flow rates employed in the study. The temperatures in the tank were continuously monitored in regions far from the jet flow to ensure that the effects due to recirculation and thermal stratification were not significant during the experiment. The jet trajectory and the downstream spread of the jet were obtained from the measured vertical temperature distributions in the jet flow at various horizontal locations. The experiments were frequently redone to evaluate the repeatability of the measurements taken. It was found that the data could be repeated to within a 5% difference. The measurements were taken over a wide range of inlet temperatures and flow rates. Three directions of the hot water discharge, horizontal and 30° and 45° downwards with the horizontal, were studied. Two locations for the discharge, one near the center of the side of the tank and the other 1.5 cm above the bottom of the tank, were considered. A detailed study of the thermal characteristics of the buoyant jet was undertaken and some of the important results are presented below.

EXPERIMENTAL RESULTS AND DISCUSSION

The experimental results are presented in terms of the dimensionless temperature ϕ defined as

$$\phi = \frac{t - t_\infty}{t_i - t_\infty} \quad (1)$$

where t is the local temperature measured at a given location in the water body, t_i is the measured inlet temperature of the jet and t_∞ the ambient fluid temperature, measured far from the buoyant jet. It is, of course, ensured that t_∞ remains unaltered during the experiment and equal to the temperature of the isothermal water body before the start of the experiment. The horizontal distance X is measured from the side of the tank where the jet opening is located and the vertical distance Y is measured vertically upwards from the center of the jet inlet. Both are nondimensionalized with D, the diameter of the jet inlet tube. The inlet conditions are given in terms of

$$Gr = \frac{g\beta D^3(t_i - t_\infty)}{\nu^2}, \quad Re = \frac{U_i D}{\nu} = \frac{4Q}{\pi D \nu} \quad (2)$$

where Re is the Reynolds number, Gr the Grashof number, U_i the inlet velocity, β the coefficient of thermal expansion of the fluid, ν the kinematic viscosity, Q the volume flow rate at the inlet and g the magnitude of the gravitational acceleration.

The hot water jet was first discharged at the central location on one side of the tank and the vertical temperature distributions were measured at various flow rates and inlet temperatures. Figure 1 shows the curves obtained at $Re = 530.6$ and $Gr = 1.6739 \times 10^4$, which give the mixed convection parameter Gr/Re^2 as 0.0594. This parameter, which determines the relative importance of natural convection in a mixed convection flow, arises when the governing momentum equation is nondimensionalized with the inlet velocity, temperature and jet diameter as characteristic quantities, as seen later. In this study, since interest lies in the mixed convection regime and not in the predominantly forced convection case of a jet or the predominantly natural convection case of a thermal plume, Gr/Re^2 has been varied in the approximate range of 0.005–0.20, which was found to be adequate in determining the main features of interest in the mixed convection regime. The buoyant jet flow entering the tank is laminar because of the low value of the Reynolds number. It is seen from Fig. 1, which shows the vertical temperature distribution at various horizontal locations, that the curves are fairly symmetric about the centerline, indicated by the peak in the profiles. The thickness or the spread of the jet may be quantitatively determined by obtaining the locations where 99% of the temperature drop from the maximum temperature to the ambient temperature has occurred. Some nonsymmetry is observed in the curves taken very near the jet inlet and the curves become symmetric as the flow proceeds downstream. The profiles are Gaussian in form and this information is used later in analysis.

It is also observed that the maximum temperature decays downstream, as is expected from the entrainment of the colder ambient fluid into the jet as the flow proceeds downstream. The thickness of the jet also increases as the jet moves away from the inlet due to this increased flow. It is evident that the height above the inlet at which maximum temperature occurs increases, as the flow proceeds downstream, indicating a vertical movement of the jet. This is expected from the buoyancy effects that arise because of the jet fluid being lighter than the ambient fluid. A comparison of the curves in Fig. 1 with those taken at a higher flow rate and, hence, at a lower value of Gr/Re^2, as shown in

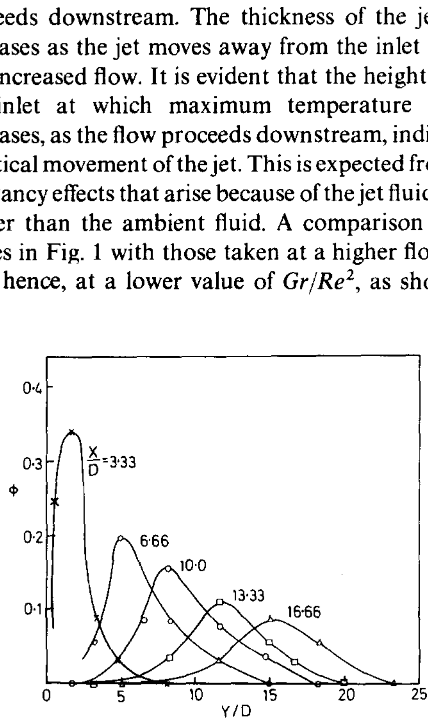

FIG. 1. Temperature profiles with inlet at the center, with $Gr/Re^2 = 0.0594$, $Re = 530.6$, $Gr = 1.6739 \times 10^4$.

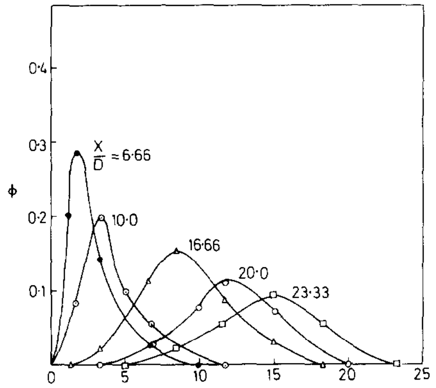

FIG. 2. Temperature profiles with inlet at the center, with $Gr/Re^2 = 0.00887$, $Re = 1376$, $Gr = 1.6739 \times 10^4$.

Fig. 2, indicates some very interesting differences. The upward shift of the jet is much smaller at the larger Re, as seen from the much lower values of Y at which the maxima in temperature occur for given values of X. The thickness of the jet is also found to be less at given downstream locations of X. This is expected for the boundary layer flow under consideration, since with increasing Re, the boundary layer thickness decreases. In these figures, the inlet temperatures are equal and it is clearly seen that the decay in the maximum, or centerline, temperature downstream is more gradual for the larger Re. This may be physically explained in terms of the larger flow velocity in this case, which allows less time for energy loss and entrainment to a fluid packet moving downstream from the inlet to a given X location. It is also seen later that the total distance travelled by the jet up to a given value of X is lower at a lower Gr/Re^2. Similar trends were observed at other inlet conditions.

The jet centerline trajectory may be obtained by plotting the location of the axis, as indicated by the location of the peak in the temperature distribution curves, on a $X-Y$ coordinate system. Figure 3 shows the jet trajectory thus obtained, by fitting a curve to the several data points with the inlet at the bottom location, as a function of Gr/Re^2. Though this parameter was varied by changing the flow rate as well as by altering the inlet temperature, Fig. 3 does not indicate any inconsistency, lending support to the argument that the results can be fairly well correlated in terms of the parameter Gr/Re^2. With increasing value of this parameter, the jet moves vertically at a faster rate, as expected from increased buoyancy effects, and the distance S traversed by the flow up to a given value of X is larger. This results in a lower centerline temperature at a given X for a larger value of Gr/Re^2, in the region far from the inlet, as seen earlier. A wide range of flow rates and inlet temperatures were considered and the trajectories obtained are shown in Fig. 3 at two values of Gr, with varying Re.

As the jet moves away from the inlet, the centerline temperature decreases and the thickness of the flow region increases because of the entrainment of the ambient fluid, which is colder and, thus, heavier than the discharged fluid. Eventually, of course, the jet meets the free surface or loses its buoyancy because of energy loss to the environment. It may also become turbulent over a sufficiently large downstream distance [8]. The decay in the centerline temperature may be determined as a function of the horizontal distance X, the vertical distance Y or the distance travelled downstream S, measured along the jet axis. For a vertical jet with a small amount of buoyancy, Mollendorf and Gebhart [5] found that the centerline temperature decays as $1/X$. The vertical buoyant jet was also studied in the present work and the centerline temperature was found to vary as $X^{-1.09}$ in one case and as $X^{-1.23}$ in the other experiment [14]. However, the theoretical work assumed a point source at the inlet, whereas we have a finite-sized jet in the present case at the inlet. A representative point source was, therefore, determined for these measurements and a fairly good agreement with theory was obtained for the vertical jet [14]. For the horizontal buoyant jet too, the temperature decay may be obtained from the graphs shown. The variation with X is obviously dependent on Gr/Re^2, though the variation with S was found to depend weakly on this parameter, indicating the general similarity in the basic processes involved in the various flow configurations.

A comparison between the experimentally obtained curves for the two inlet locations, central and bottom, is made in Fig. 4 at two values of Gr/Re^2. The curves for the two circumstances are found to be quite close. The vertical movement of the jet for the central discharge is found to be slightly more rapid. This leads to a larger distance of traverse S for the jet and, hence, somewhat lower centerline temperatures, as confirmed from the results obtained. However, the difference is not large and may be attributed to the proximity of the tank bottom to the lower inlet, which tends to pull the jet downwards due to reduced entrainment from below

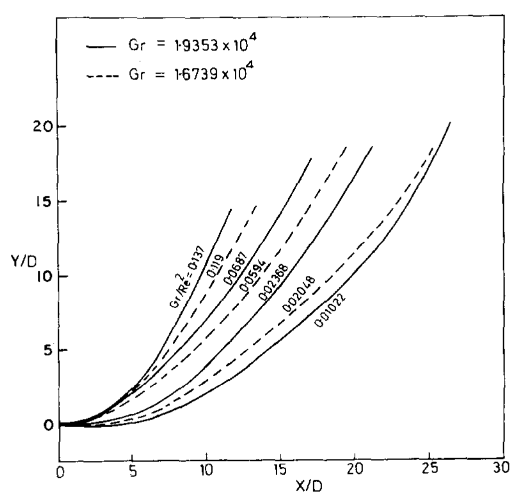

FIG. 3. Jet centerline trajectory with inlet at the bottom.

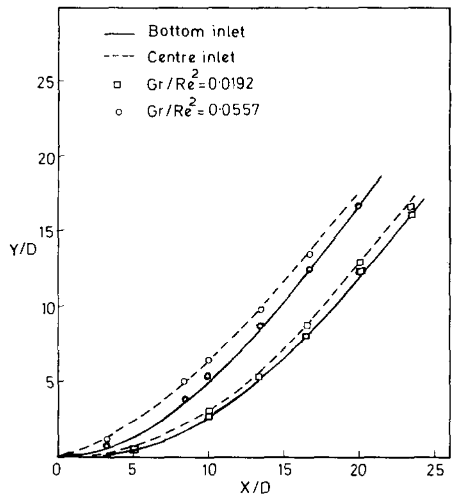

FIG. 4. Comparison of the jet centerline trajectory with inlet at the bottom and at the center.

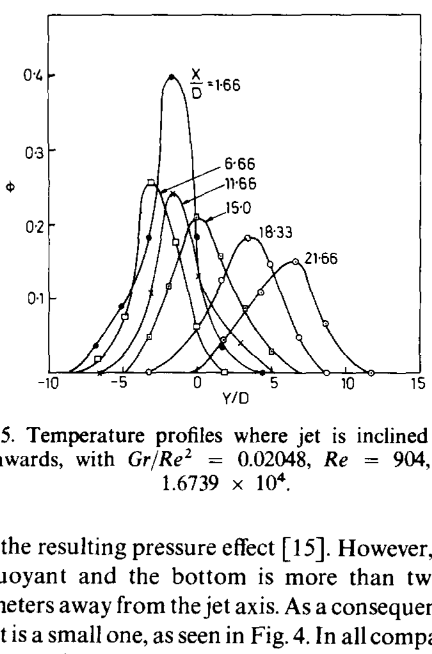

FIG. 5. Temperature profiles where jet is inclined at 30° downwards, with $Gr/Re^2 = 0.02048$, $Re = 904$, $Gr = 1.6739 \times 10^4$.

and the resulting pressure effect [15]. However, the jet is buoyant and the bottom is more than two inlet diameters away from the jet axis. As a consequence, the effect is a small one, as seen in Fig. 4. In all comparisons of the results for the two inlet locations, the difference was small and generally of the order of the experimental error, in regions far from the inlet. As such, the results for the horizontally discharged jet are seen to be mainly dependent on the parameter Gr/Re^2, which is, therefore, the governing parameter for this flow.

A study of buoyant jets discharged at an inclination with the horizontal was also carried out. Inclinations of 30° and 45° downwards were considered. This was achieved experimentally by making an inclined hole on a Perspex sheet of about 1 cm thickness and then fitting it to the inside wall of the tank at the central hole. The jet of water then enters the tank at an inclination of 30° or 45° with the horizontal. Temperature distributions at various horizontal locations in the central plane of the jet were obtained. The measured temperature profiles for the 30° inclination are shown at $Re = 904$ and $Gr = 1.6739 \times 10^4$ in Fig. 5. In these curves, a peak in the profiles is observed as before. However, the uniform shift of the location of the peak to larger Y is not observed in this case. In this figure, the peaks at X/D values of 1.66, 6.66 and 11.66 occur at negative Y indicating a downward movement of the jet. From $X/D = 1.66$ to $X/D = 6.66$, the peak has moved downwards and then it moves upwards. At $X/D = 15.0$, the jet axis is essentially at the same height as the jet inlet and the jet continues to move vertically upwards as seen before for the horizontal jet, with the peaks moving to larger Y values.

In the case of an inclined buoyant jet, the initial momentum is at an angle with the horizontal and the jet moves downwards under its action. However, the buoyancy force acts vertically upwards and, under the action of these two effects, the downward velocity of the jet becomes zero after a certain downward movement and then the jet moves upwards. Similar curves are obtained for the 45° inclination, as shown in Fig. 6. A comparison between the curves shown in Figs. 5 and 6 indicates a more delayed upward movement for the 45° inclination. In this case the discharged jet reaches the horizontal plane of the inlet at an X/D value of about 17.0. Similarly, the peaks in the profiles at X/D values of 18.33 and 21.66 also occur at lower values of Y. This implies that, for the 45° inclination, the jet goes further down, as compared to the 30° discharge, before starting on the upward movement. This is clearly seen in Fig. 7 where the trajectories are shown for three values of Gr/Re^2 and for these two inclinations. The trends observed here are therefore physically expected.

It is also seen that if Gr/Re^2 is decreased, which may be achieved by increasing the flow rate or by decreasing the inlet temperature, the jet proceeds further down vertically before moving up and crosses the horizontal plane through the inlet location at a larger value of X. With an increased flow rate or decreased inlet temperature, the buoyancy effects are lower and the jet covers a larger horizontal and vertical distance before turning upwards. A similar behavior is observed when the jet inclination is increased. The decay of the

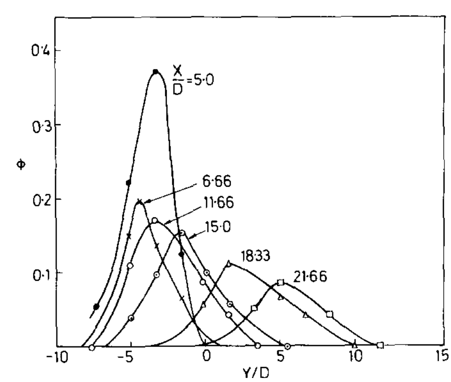

FIG. 6. Temperature profiles with jet inclined at 45° downwards, with $Gr/Re^2 = 0.02048$, $Re = 904$, $Gr = 1.6739 \times 10^4$.

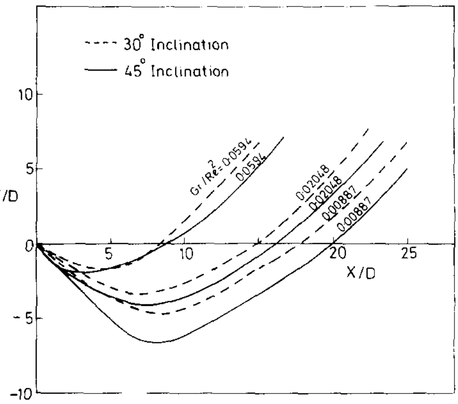

FIG. 7. Jet centerline trajectory with different inclinations.

centerline temperature with X, or with the distance along the jet axis S, may also be determined from the results shown. It is again seen that the centerline temperature is lower if the jet traverses a larger distance S. The trajectory indicates the nature of the anticipated temperature field and the measurements agree with the physical aspects of the flow.

ANALYSIS

The flow under consideration is a 3-dim. mixed convection circumstance. The main flow direction being inclined with the vertical buoyancy force, axisymmetry does not arise and the buoyancy force has components along the jet axis and also perpendicular to it. As a consequence, an exact solution of the problem demands a consideration of the coupled momentum, continuity and energy equations. This is obviously a very involved problem to be solved analytically. However, the experimental results obtained may be employed to develop a simple analytical model to predict the essential features of the flow.

It has been observed that the measured temperature profiles are similar in form at various horizontal locations. The profiles are Gaussian in form, with the jet centerline indicated by the vertical location of the maximum temperature at a given horizontal position. The centerline temperature decays with X and with the distance S along the jet axis. The governing parameter was found to be Gr/Re^2, where both Gr and Re are based on the inlet tube diameter D and the inlet average velocity and temperature. In addition to these observations, it may also be noted that the tube leading to the inlet being long, the flow in the tube may be taken as a fully-developed Poiseuille flow, with a parabolic velocity distribution. Since the tube is well insulated between the heating bath and the discharge location, the temperature at the tube exit may be assumed to be uniform.

The analytical model for the flow under consideration is based on the integral method employed for various free boundary flows [8, 16, 17]. The velocity and temperature profiles are assumed to be Gaussian and are given by

$$u = U_c \exp(-r^2/R^2), \quad t - t_\infty = (t_c - t_\infty)\exp(-r^2/A^2R^2) \tag{3}$$

where U_c and t_c are the centerline velocity and temperature respectively, u and t are the local velocity and temperature, r is the radial distance from the jet axis taken normal to it, R is the jet width and A is the ratio of the spread of the temperature field to that of the velocity field. The jet width R is given by the radial location where the velocity drops to 1/e times the value at the centerline, where e is the exponential. The entrainment determines the spreading of the jet and the entrainment velocity at the edge of the jet is taken as proportional to the centerline velocity. Therefore,

$$\frac{d}{dS}\left[\int_0^\infty u\,2\pi r\,dr\right] = 2\pi\alpha R U_c. \tag{4}$$

The entrainment coefficient α as given above is related to that for the top-hat profiles α_h, frequently employed in turbulent flows, as $\alpha = \alpha_h/\sqrt{2}$ [18].

Similarly, the equations for the conservation of momentum, horizontal and vertical, and of energy may be written. Employing the Boussinesq approximation for the density difference, these equations are obtained in terms of the path length S along the jet axis as

$$\frac{d}{dS}\left[\int_0^\infty \rho u^2 \cos\theta\, 2\pi r\,dr\right] = 0, \tag{5}$$

$$\frac{d}{dS}\left[\int_0^\infty \rho u^2 \sin\theta\, 2\pi r\,dr\right] = \int_0^\infty g\beta\rho(t - t_\infty)2\pi r\,dr, \tag{6}$$

$$\frac{d}{dS}\left[\int_0^\infty \rho u C_p(t - t_\infty)2\pi r\,dr\right] = 0, \tag{7}$$

$$\frac{dX}{dS} = \cos\theta, \quad \frac{dY}{dS} = \sin\theta \tag{8}$$

where θ is the angle of inclination of the jet axis with the horizontal, C_p is the specific heat of the fluid and ρ its density. Therefore, u and $(t - t_\infty)$ from equation (3) may be substituted in the above equations and the integrations carried out. Then the differential equations for U_c, $(t_c - t_\infty)$, R and θ would be obtained.

Employing the nondimensionalization given earlier, the dimensionless variables, denoted by primes, are given by

$$S' = S/D, \quad X' = X/D, \quad Y' = Y/D,$$

$$U' = u/U_i, \quad R' = R/D, \quad \phi = (t - t_\infty)/(t_i - t_\infty). \tag{9}$$

Therefore, the centerline velocity and temperature are denoted by U'_c and ϕ_c. The governing equations yield the following differential equations for the above dimensionless variables:

$$\frac{dU'_c}{dS'} = 2A^2 \frac{Gr}{Re^2} \frac{\phi_c}{U'_c} \sin\theta - \frac{2\alpha U'_c}{R'}, \tag{10}$$

$$\frac{dR'}{dS'} = 2\alpha - A^2 \frac{Gr}{Re^2} \frac{\phi_c}{(U'_c)^2} \sin\theta, \tag{11}$$

$$\frac{d\theta}{dS'} = 2A^2 \frac{Gr}{Re^2} \frac{\phi_c}{(U'_c)^2} \cos\theta, \tag{12}$$

$$(R')^2 U_c \phi_c = \text{constant}, \tag{13}$$

$$\frac{dX'}{dS'} = \cos\theta, \quad \frac{dY'}{dS'} = \sin\theta. \tag{14}$$

These are coupled first order ordinary differential equations and may be solved numerically by various shooting methods available to determine the flow characteristics of interest. The jet trajectory is obtained by determining X and Y. For equation (13) the conditions at the inlet are employed for determining the constant. The entrainment model employed here is based on the work of Morton *et al.* [16]. Other entrainment models may also be considered, particularly that developed by Morton [17], for laminar flows. However, this model leads to the emergence of Re as an independent parameter and is largely applicable for vertical flows generated by a point source. Our experiment has clearly indicated the dominance of Gr/Re^2 as the governing parameter, instead of Gr or Re, and the flow is due to a finite-size inlet. Moreover, the flow is inclined with the vertical and the entrainment characteristics are quite different from the vertical case. As a consequence, the former entrainment model is employed to study the basic features of the buoyant jet flow. Similarly, more accurate integral method analyses may be considered. We present here some results from the above approximate model.

The conditions at the inlet are given in terms of the average velocity U_i, for the parabolic distribution of a fully-developed pipe flow, and temperature t_i, which is uniform across the tube cross-section. Therefore, the inlet flow rate, momentum input and energy input may be determined. Gaussian profiles at the inlet are determined so that these inputs into the flow are the same. Therefore, the starting conditions for equations (10)–(14) are

$$\text{at } S' = 0, \quad X' = Y' = \theta = 0,$$

$$U'_c = (U'_c)_G, \quad \phi_c = (\phi_c)_G, \quad R' = (R')_G \tag{15}$$

where the subscript G refers to the corresponding values for the assumed Gaussian profiles at the inlet. Therefore, the inlet mass flow rate, momentum input and energy input were preserved by the assumed profiles.

The governing parameters are obtained from equations (10)–(14) as Gr/Re^2, α and A. The parameter A compares the spread of the thermal field with that of the velocity field. For a buoyant jet emerging from a point source, this parameter will be of order $1/Pr^{1/2}$ where Pr is the Prandtl number of the fluid. However, in the present case, the spread of the thermal and the velocity fields is of the same order at the inlet and, as the flow proceeds downstream, A is expected to deviate from around 1.0, attaining a value of around $1/Pr^{1/2}$ far from the inlet, if the flow continues to be laminar. Such a variation in A may easily be built into the analysis. But information on the velocity field is also needed to specify A accurately. We have carried out the solution of the problem taking A as unity, which applies at the inlet, and have discussed the effect of a variation in A further downstream on the results obtained. The parameter α was also varied from 0.05 to 0.15 and it was found that the results are rather insensitive to its value. A value of 0.07 was chosen, which corresponds to a value of 0.10 for top-hat profiles and which has been employed in several studies of buoyant jet and plume flows.

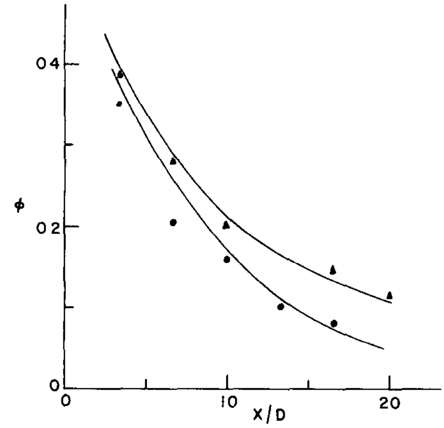

FIG. 8. Comparison between theoretical and experimental results for the decay of centerline temperature with X. Data: ▲, $Gr/Re^2 = 0.00887$; ●, $Gr/Re^2 = 0.0594$.

The numerical results obtained indicated the basic features observed experimentally for the horizontal jet and for the other inclinations. A comparison of the theoretical results with a few experimental results is shown for the horizontal jet in Figs. 8 and 9. In Fig. 8,

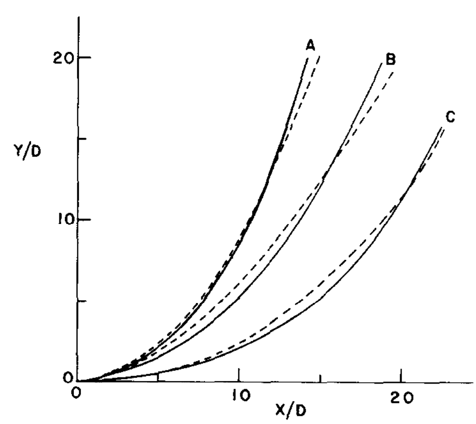

FIG. 9. Comparison between the theoretical and experimental results for the centerline trajectory. A, $Gr/Re^2 = 0.119$; B, $Gr/Re^2 = 0.0594$; C, 0.02048. ——, theoretical; – – –, experimental.

the decay of the centerline temperature with X is compared for $Gr/Re^2 = 0.0594$ and 0.00887. A fairly good agreement is observed for both the curves, lending support to the analytical model employed. A comparison between the predicted trajectory and that obtained experimentally, by fitting a curve to the data points obtained for the centerline, is shown in Fig. 9 for $Gr/Re^2 = 0.119$, 0.0594 and 0.02048. The experimental curves are obtained from Fig. 3. Again, a fairly good agreement is observed. It is seen in all the cases shown that the predicted curve tends to rise faster than the experimental one in the region far from the inlet. This is due to the thermal region being thinner than the velocity region away from the inlet. This implies a lower value of A as the flow moves away from the inlet and, therefore, a less rapid vertical movement of the flow, as may be seen from the governing equations. Near the inlet, the thermal region is comparable to the velocity region and this results in a more gradual rise for the predicted profiles. Therefore, if information is available on the velocity field, the variation of A downstream may be chosen suitably to yield a closer agreement between theory and experiment.

The analytical model was also employed for the jet inclined with the horizontal and, again, a fairly good agreement was obtained. The governing parameter is Gr/Re^2, the Prandtl number dependence appearing through the parameter A. Therefore, this simple model indicates the basic features of the flow and the governing physical processes that determine the flow trajectory and characteristics downstream. The centerline temperature and velocity vary with the path length S and the values obtained at a given X would depend on the distance travelled by the jet to that position.

CONCLUSIONS

An experimental study of the thermal characteristics of a laminar buoyant jet discharged horizontally, and at an inclination with the horizontal, into an extensive isothermal ambient medium is carried out, followed by an analytical study of the flow. The jet trajectory is determined and its dependence on the direction of the inflow, rate of flow and on the inlet temperature is investigated. The decay of the centerline temperature of the jet and the increase in jet thickness downstream are examined for various inflow conditions. When the fluid is discharged at a downward inclination with the horizontal, the jet initially moves downwards, the downward velocity ultimately coming to zero and the jet moving vertically upwards. The downward penetration of the jet, before it turns upward, is found to increase with an increase in the inflow inclination, with a decrease in inflow temperature and with an increase in inlet flow rate. The results are found to depend largely on the mixed convection parameter Gr/Re^2, which is based on the inlet conditions. An analytical model, employing the integral method approach, is developed on the basis of the experimental observations. The theoretical results are found to be in fairly good agreement with the experimental results, indicating the validity of such an analytical model for predicting the jet trajectory and the downstream behavior of the flow.

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ETUDE DE JETS LAMINAIRES NATURELS INCLINES PAR RAPPORT A LA VERTICALE

Résumé—On étudie expérimentalement la trajectoire et d'autres caractéristiques thermiques d'un jet libre incliné par rapport aux forces d'Archimède. Un jet d'eau chaude est injecté, horizontalement ou avec inclinaison par rapport à l'horizontale, dans l'eau qui simule un milieu ambiant isotherme. La température dans le jet est mesurée et la trajectoire du jet sous l'action des forces externes et de la quantité du mouvement est étudiée en fonction de la vitesse et de la température à l'entrée. Pour les jets inclinés qui s'écoulent vers le bas, l'étude détermine la pénétration du jet. La diminution de la température sur la ligne de centre et l'accroissement de l'épanouissement du jet sont mesurés. Les résultats expérimentaux sont obtenus pour différentes valeurs du paramètre de convection mixte lequel unifie correctement les comportements observés. Les résultats sont aussi considérés en fonction des mécanismes physiques qui gouvernent l'écoulement et un modèle analytique simple, basé sur l'approche intégrale, est développé pour généraliser les observations expérimentales. La trajectoire calculée et la décroissance de la température sur la ligne centrale sont en accord avec les résultats expérimentaux.

UNTERSUCHUNG EINES LAMINAREN AUFTRIEBS-STRAHLS, DER UNTER EINEM BESTIMMTEN WINKEL ZUR SENKRECHTEN AUFTRIEBSKRAFT STRÖMT

Zusammenfassung—Ausführliche experimentelle Untersuchungen wurden durchgeführt, um die Ausbreitung und andere thermische Eigenschaften eines Auftriebs-Strahls zu untersuchen, der in einem bestimmten Winkel zu der senkrechten Auftriebskraft zu strömen beginnt. Ein Strahl aus heißem Wasser wird (waagerecht und mit verschiedenen Winkeln zur Waagerechten) in einen Wasserbehälter injiziert, der eine ausgedehnte isotherme Umgebung simulieren soll. Die Temperaturverteilung im Strahl wird bestimmt. Die Ausbreitung des Strahls unter der Einwirkung der Auftriebskraft und des Anfangsimpulses wird in Abhängigkeit von der Einströmgeschwindigkeit und der Temperatur untersucht. Für den nach unten gerichteten Strahl ergibt die Untersuchung das entsprechende Eindringvermögen. Die Abnahme der Kerntemperatur und die zunehmende Ausbreitung des Strahls werden gemessen. Man erhält experimentelle Ergebnisse für verschiedene Werte des Misch-Konvektions-Parameters, der die beobachteten Trends sehr gut korreliert. Die Ergebnisse werden auch anhand der grundlegenden physikalischen Mechanismen, die die Strömung bestimmen, betrachtet. Ein einfaches analytisches Modell, das auf einer integralen Näherung beruht, wird entwickelt, um die experimentellen Beobachtungen zu verallgemeinern. Die damit berechnete Ausbreitung und Abnahme der Kerntemperatur zeigen eine recht gute Übereinstimmung mit den Versuchsergebnissen.

ИССЛЕДОВАНИЕ ЛАМИНАРНЫХ СВОБОДНОКОНВЕКТИВНЫХ СТРУЙ, ИСТЕКАЮЩИХ ПОД УГЛОМ К ВЕРТИКАЛЬНО НАПРАВЛЕННОЙ ПОДЪЕМНОЙ СИЛЕ

Аннотация—Проведено подробное экспериментальное исследование траектории и тепловых характеристик свободноконвективной струи, истекающей под углом к вертикально направленной подъемной силе. Струя горячей воды подается горизонтально и под разными углами в объем воды, моделирующий протяженную изотермическую окружающую среду. Определено распределение температуры в струе, а траектория струи, на которую оказывает влияние подъемная сила и начальный импульс, исследуется как функция скорости истечения струи и температуры. Для наклонных направленных вниз струй определяется глубина их проникновения. Измерены уменьшение температуры по оси струи и увеличение размаха струи вниз по течению. Получены экспериментальные данные для различных значений параметра смешанной конвекции, который, как установлено, очень хорошо обобщает полученные результаты. Рассмотрены также основные физические механизмы, определяющие течение струй, и для обобщения экспериментальных данных разработана простая аналитическая модель, основанная на интегральном методе. Расчетные значения траектории и уменьшения температуры по оси струи хорошо согласуются с экспериментальными результатами.